Influence of partially known parameter on flaw characterization in Eddy Current Testing by using a random walk MCMC method based on metamodeling

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Abstract. First, we present the implementation of a random walk Metropolis-within-Gibbs (MWG) sampling method in flaw characterization based on a metamodeling method. The role of metamodeling is to reduce the computational time cost in Eddy Current Testing (ECT) forward model calculation. In such a way, the use of Markov Chain Monte Carlo (MCMC) methods becomes possible. Secondly, we analyze the influence of partially known parameters in Bayesian estimation. The objective is to evaluate the importance of providing more specific prior information. Simulation results show that even partially known information has great interest in providing more accurate flaw parameter estimations. The improvement ratio depends on the parameter dependence and the interest shows only when the provided information is specific enough.
Key words : flaw characterization, Eddy Current Testing, random walk, Markov Chain Monte Carlo (MCMC), metamodeling.

1. Introduction

In Eddy Current Testing (ECT), the objective is to estimate the parameters of the flaws present in the specimen examined, like their positions, dimensions and parameters related to their shapes, etc. In our situation, the processed data is the variation of impedance caused by the presence of a flaw.

So, if we denote $\boldsymbol{y} \in \mathbb{C}^M$ as the impedance variation where M is the measurment dimension, $\boldsymbol{x} \in \mathbb{R}^N$ as flaw parameters where N is the unknown parameter dimension, without considering the measurement noise, they can be related to each other by

$$\boldsymbol{y} = f(\boldsymbol{x}). \tag{1}$$

Here, $f(\boldsymbol{x})$ represents the physical phenomenon from $\boldsymbol{x}$ to $\boldsymbol{y}$. Obviously, it is not linear. The objective of flaw characterization is to estimate $\boldsymbol{x}$ from measurements of $\boldsymbol{y}$, of course noisy. One of the major difficulties in ECT is the numerical realization of the mathematical description of the physical phenomenon represented by $f(\boldsymbol{x})$. The method of moments [1, 2, 3] allows to get an accurate modeling of this phenomenon. However, it is computationally costly. This, in the inverse problem, limits the use of more sophisticated stochastic methods, like Maximum Likelihood (ML), Maximum A Posteriori (MAP) and Expected A Posteriori (EAP), since these methods are often solved by iterative numerical algorithms [4, 5] or Markov Chain Monte Carlo (MCMC) sampling methods [6, 7, 8]. Those require to calculate $f(\boldsymbol{x})$ many times.

A metamodeling method [9, 10, 11] provides surrogate options to get $f(\boldsymbol{x})$ with lower time cost. In metamodeling, this includes

- a well-trained database containing samples of $\boldsymbol{x}$ and the corresponding $f(\boldsymbol{x})$;
- an interpolator which allows to predict $f(\boldsymbol{x})$ for any input $\boldsymbol{x}$.

Bilicz *et al.* [12] present an *optimal* database training process by using the kriging interpolation. The kriging interpolation method [11, 13, 14] considers the interpolated samples as a Gaussian process guided by a covariance matrix. Interpolations minimizing the error distance have analytical expressions. Applying metamodeling in flaw characterization problem, the database only needs to be trained once at the beginning. When a forward calculation is required, it only needs to perform the kriging interpolation, which is much less costly computationally. For this reason, it makes the use of MCMC methods possible in ECT flaw characterization problem.

The interest of using MCMC sampling method here is first to get EAP estimations of flaw parameters, second, to calculate the variance of posterior distribution, third, to analyze the influence of a given parameter on other parameter estimations. Similar works [15, 16] have been conducted in view of parameter estimation. We are going to focus on the influence of partially known information. This is managed by the means of the MCMC sampling method.

2. Parameter estimation using random walk MCMC method

2.1. Likelihood with Gaussian noise model

Considering Gaussian noise present in the measurements, the forward model can be represented by

$$\boldsymbol{y} = f(\boldsymbol{x}) + \boldsymbol{\epsilon}, \text{ with } \boldsymbol{\epsilon} \sim \mathcal{N}(0, \Sigma). \tag{2}$$

$f(\boldsymbol{x})$ is the non-linear model describing the relation between $\boldsymbol{x}$ and $\boldsymbol{y}$. In our situation, it is replaced by the metamodel database and the *kriging* interpolator. $\Sigma = ||\boldsymbol{\sigma}_y||^2 \mathbf{I}_M$ is the covariance matrix of the complex Gaussian noise. $\mathbf{I}_M$ is an identity matrix of size $M \times M$. The real and imaginary parts of $\boldsymbol{\epsilon}$ both follow Gaussian distributions. Their variances are $\sigma_R^2 = \sigma_I^2$. The standard deviation $\boldsymbol{\sigma}_y = \sigma_R + j\sigma_I$.

The likelihood of such a complex multivariate Gaussian model can then be given by

$$p(\boldsymbol{y}|\boldsymbol{x}) = \frac{1}{(\pi\sigma^2)^M} \exp\Big\{ -\frac{1}{\sigma^2} ||\boldsymbol{y} - f(\boldsymbol{x})||^2 \Big\}, \tag{3}$$

where $\sigma^2 = ||\boldsymbol{\sigma}_y||^2$. In our situation, it is calculated based on the data Signal-to-Noise Ratio (SNR).

2.2. Uniform prior model

As for the parameters of the flaw, we choose to use the following uniform distribution

$$\boldsymbol{x} \sim \mathcal{U}(\boldsymbol{a}, \boldsymbol{b}) : \ p(\boldsymbol{x}) = \prod_{i=1}^{N} p_i(x_i), \ p_i(x_i) = \begin{cases} \frac{1}{b_i - a_i} & a_i \le x_i \le b_i \\ 0 & \text{else} \end{cases}, \ i = 1, 2, \cdots, N. \tag{4}$$

$\boldsymbol{a} = [a_1, a_2, \cdots, a_N]^T$ and $\boldsymbol{b} = [b_1, b_2, \cdots, b_N]^T$ denote the inferior and the superior borders for each parameter in the metamodel database. Denote $\mathbb{U}^\circ$ as the support for $\boldsymbol{x}$, it is bounded by the inferior and superior borders $\boldsymbol{a}$ and $\boldsymbol{b}$.

2.3. EAP estimation

Bayesian methods impose prior distributions about flaw parameters to estimate their values from noisy measurements. The EAP estimation calculates the expectation of the posterior distribution which can be described by

$$\hat{\boldsymbol{x}} = \int_{(\boldsymbol{x}\in\mathbb{U}^\circ)} \boldsymbol{x}\; p(\boldsymbol{x}|\boldsymbol{y})\; \mathrm{d}\boldsymbol{x} = \int_{(\boldsymbol{x}\in\mathbb{U}^\circ)} \boldsymbol{x}\; \frac{p(\boldsymbol{y}|\boldsymbol{x})p(\boldsymbol{x})}{p(\boldsymbol{y})}\; \mathrm{d}\boldsymbol{x}. \tag{5}$$

where $p(\boldsymbol{y}|\boldsymbol{x})$ is the likelihood given in Eq. (3). $p(\boldsymbol{x})$ is the prior distribution given in Eq. (4). $p(\boldsymbol{y})$ is the residual which is independent of $\boldsymbol{x}$.

2.4. Random walk MCMC

In order to get the estimations of the posterior distribution $p(\boldsymbol{x}|\boldsymbol{y})$, we choose to use the following random walk Metropolis-Within-Gibbs (MWG) method as described in Tab. 1. It is a combination of the random walk Hasting-Metropolis method [6, 17, 18] and the Gibbs alternating sampling strategy [7, 19].

Table 1: scheme of random walk Metropolis-within-Gibbs method. K is the sampling number.

1). Initiation: $\hat{\boldsymbol{x}}^{(k)}$, $k=0$, $i=1$.

2). Alternating candidature proposition: $\hat{\boldsymbol{x}}^{(p)} = \hat{\boldsymbol{x}}^{(k)}$**,**

- 2.1, **alternating parameter** $\hat{x}_i^{(p)}$:
  - generate randomly $s_i \sim \mathcal{U}(-0.5t_i, 0.5t_i)$, $\hat{x}_i^{(p)} = \hat{x}_i^{(k)} + s_i$.
  - *if* $(i=N)$ $i=1$, *else* $i++$;
- 2.2, **verification:**
  - *if* $(\hat{x}_i^{(p)} \in [a_i, b_i])$ go to **3** , *else* repeat 2.1 .

3). Decision:

if $\left[r = \frac{p(\hat{\boldsymbol{x}}^{(p)}|\boldsymbol{y})}{p(\hat{\boldsymbol{x}}^{(k)}|\boldsymbol{y})} > 1\right]$ accept $\hat{\boldsymbol{x}}^{(p)}$, $\hat{\boldsymbol{x}}^{(k+1)} = \hat{\boldsymbol{x}}^{(p)}$,

else

3.1, generate $u \sim \mathcal{U}(0,1)$**;**
3.2, sub-decision: *if* $(u<r)$ accept $\hat{\boldsymbol{x}}^{(p)}$, $\hat{\boldsymbol{x}}^{(k+1)} = \hat{\boldsymbol{x}}^{(p)}$, *else* reject $\hat{\boldsymbol{x}}^{(p)}$, $\hat{\boldsymbol{x}}^{(k+1)} = \hat{\boldsymbol{x}}^{(k)}$.

$k++$.

4). Repeat 2, 3 until $k = K$**.**

Once applied the sampling method, we get a set of samples $\{\hat{\boldsymbol{x}}^k\}_K$ for $k = 1, 2, \cdots, K$ following the posterior distribution $p(\boldsymbol{x}|\boldsymbol{y})$ after a period of warm-up (or burn-in) time. Then the EAP estimation given in Eq. (5) can be approximated by averaging over all the samples:

$$\hat{\boldsymbol{x}} = \int_{(\boldsymbol{x}\in\mathbb{U}^\circ)} \boldsymbol{x}\; p(\boldsymbol{x}|\boldsymbol{y})\; \mathrm{d}\boldsymbol{x} \approx \frac{1}{K}\sum_{k=1}^{K} \hat{\boldsymbol{x}}^{(k)}. \tag{6}$$

To appraise the corresponding estimation uncertainty, the variance of the generated samples can be used. It is defined by

$$\boldsymbol{\sigma}_p^2 = D\big[\boldsymbol{x}\big] = E\Big[\big(\boldsymbol{x} - E[\boldsymbol{x}]\big)^2\Big] \tag{7}$$

It corresponds to the variances of the posterior distribution along different parameters. $E(\boldsymbol{x})$ can be replaced by its EAP estimation given in Eq. (6), then $\boldsymbol{\sigma}_p$ can be approximated by

$$\boldsymbol{\sigma}_p \approx \sqrt{\frac{1}{K-1}\sum_{k=1}^{K}\left(\hat{\boldsymbol{x}}^{(k)} - \hat{\boldsymbol{x}}\right)^2}. \tag{8}$$

3. Parameter estimation with partially known information

In certain situations, more specific prior information can be obtained. For example, the lift-off of the coil is often considered to be known while the width of a surface crack is often known to be small. The interest of studying the influence of a partially known parameter is to evaluate the importance of such kind of partial prior information, more importantly, the influence on other parameter estimations. Here, we define:

Partially known: *if the parameter support* $\boldsymbol{x} \in \mathbb{U}$ *is smaller than the metamodel database support* $\mathbb{U}^{\circ}$*, which means* $\mathbb{U} \subset \mathbb{U}^{\circ}$*, then the parameter* $\boldsymbol{x}$ *is said to be partially known.*

According to this definition, two situations might happen. One is that certain elements of $\boldsymbol{x}$ are totally known, the other is that the support for certain parameters are subsets of $\mathbb{U}^{\circ}$. In situations with partially known parameters, the same MWG method can be used to solve the estimation problem. It is only necessary to eliminate the known parameters from $\boldsymbol{x}$ or reset the inferior and superior borders to the corresponding borders in $\mathbb{U}$.

4. ECT example

4.1. Ferromagnetic configuration and data simulation

In order to analyze the influence of partially known parameters, we use a simple example as sketched in Fig. 1. An opening crack affects the surface of a conductive plate. The crack center is located at the origin O. We wish to characterize the dimension of the crack. There are three unknown parameters in our problem: flaw depth d, length l and width w.

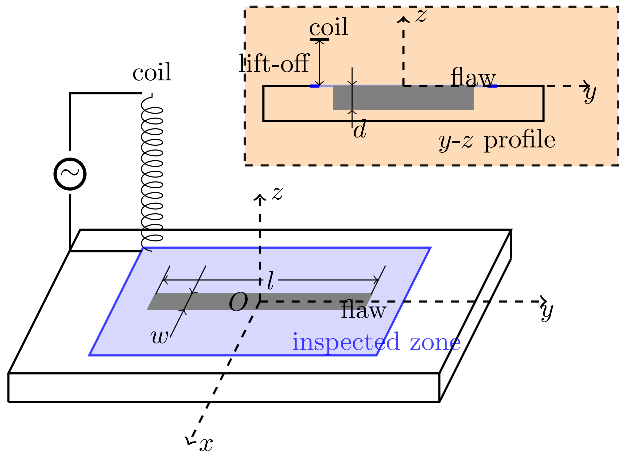


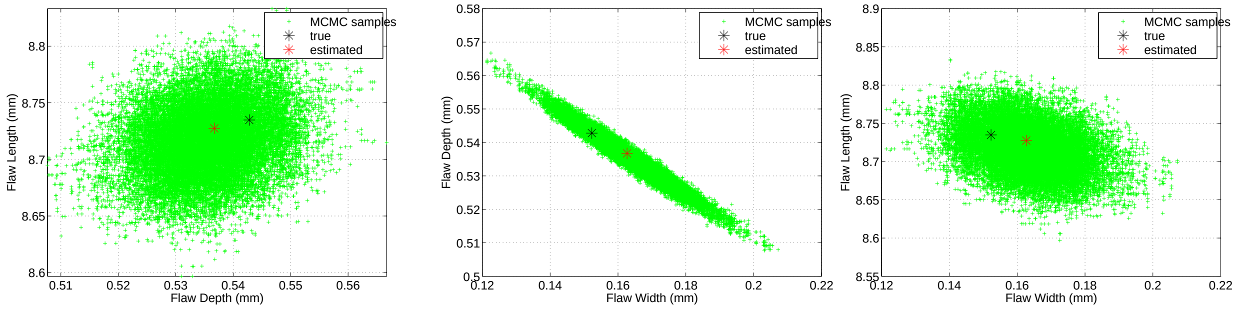
Figure 1: a 3D testing example: a flaw (in gray) at the surface of a conductive plate.

coil	
inner radius	1.0 mm
outer radius	1.75 mm
length	2.0 mm
number of turns	328
lift-off	0.303 mm
frequency	100 kHz
metal plate	
thickness	1.55 mm
conductivity	1.02 MS/m
inspected zone	
along x	$[-10, 10]$ mm
along y	$[-15, 15]$ mm
displacement step	0.5 mm

For the following analysis, the noise-free data is simulated by using the metamodel trained from simulated data using the method of moments implemented in CIVA [20]. Gaussian noise with SNR equal to 20 dB is then added numerically as described in Eq. (2). The testing sample is taken arbitrarily by using Latin Hypercube Sampling (LHS) method. Its parameter values are $d = 0.5428$ mm, $l = 8.7348$ mm and $w = 0.1523$ mm.

4.2. All unknown

First, we apply the Bayesian inverse method based on random walk MWG as described in Sec. 2 on the simulated data to estimate the three unknown parameters. Fig. 2 shows the MCMC samplings according to different profile views and the estimated results after eliminating warm-up samplings (around 700 out of 10^5 in total). In terms of CPU time, it takes around 4 minutes on a 3.4 GHz PC.



(a) d-l profile view. (b) w-d profile view. (c) w-l profile view.

Figure 2: MCMC samples ($K = 10^5$, walk step $\boldsymbol{t} = [0.01, 0.01, 0.04]$) of random walk MWG method for tested sample $d = 0.5428$ mm, $l = 8.7348$ mm, $w = 0.1523$ mm. The estimated results are $\hat{d} = 0.5385 \pm 0.0122$ mm, $\hat{l} = 8.7310 \pm 0.1739$ mm and $\hat{w} = 0.1597 \pm 0.0110$ mm.

We observe that the relative estimation error is less than 1.3% for flaw depth and length and around 4.9% for flaw width. From the w-d profile view, the EAP estimation is located in a narrow valley. This indicates that there is a determination difficulty between flaw width and depth. In other words, the flaw depth and width are highly dependent. In contrast, flaw length is less dependent on the other two parameters.

4.3. Partially known flaw width

In the above situation, we consider that all parameters are totally unknown. Strictly speaking, this is not correct, since we have benefited from prior information to get parameter inferior and superior borders in training the metamodeling database. But considering that the database support $\mathbb{U}^{\circ}$ often covers a much larger range than the one of our interest, so it can still be seen as *all unknown* situation.

Table 2: influence of partially known flaw width on estimations of the flaw depth and length.

Information level		**flaw depth**		**flaw length**	
		$\lvert\frac{\hat{\boldsymbol{x}}-\boldsymbol{x}}{\boldsymbol{x}}\rvert$ (%)	$\boldsymbol{\sigma}_p$ (mm)	$\lvert\frac{\hat{\boldsymbol{x}}-\boldsymbol{x}}{\boldsymbol{x}}\rvert$ (%)	$\boldsymbol{\sigma}_p$ (mm)
all unknown		0.7834	0.0122	0.0436	0.1739
partially known width	5	0.7864	0.0120	0.0426	0.1742
confidence radius r_c	2.5	**0.6777**	**0.0114**	**0.0434**	**0.1714**
(in times of $\hat{\sigma}_p = 0.011$ for flaw width	1.5	**0.2289**	**0.0111**	**0.0368**	**0.1685**
in *all-unknown* situation)	1	**0.1438**	**0.0099**	**0.0287**	**0.1644**
known width $w = 0.1523$		0.1114	0.0084	0.0261	0.1611

To study the influence of partially known parameters, we choose to give more prior information to the flaw width. As we observe from Fig. 2, the flaw width is highly dependent on the flaw depth but lightly on the flaw length. This makes it becomes a very good object to study influence of partial information. We define r_c as the radius of the confidence region for flaw width. It is the half distance from the inferior border to the superior border of $\mathbb{U}$ but given in ratio to the posterior standard deviation estimated in *all-unknown* situations. To simplify the simulation, $\mathbb{U}$ defined by r_c is always centered at the true value. So when $r_c = 3$, it means that more than 90% samples obtained in the *all-unknown* situation are located within the new support $\mathbb{U}$. Tab. 2 shows the estimation errors and standard deviations with partially known information.

We see that the estimation relative error and standard deviation start to decrease when r_c becomes less than 3. For flaw depth, an improvement of 85% on relative distance and 31% on standard deviation is achieved continuously while narrowing the confidence region. As for the flaw length, the improvement ratio is not as significant as for the depth. The improvement ratio is 40% for relative distance and 7% for standard deviation. This is because the flaw width is highly dependent on the flaw depth but less dependent on the flaw length, as seen from Fig. 2.

5. Conclusion and perspectives

This work analyzes the influence of partially known parameters on estimation of other unknown parameters. It is carried out by using a random walk MCMC sampling method based on a fast surrogate forward model. From the simulation results, we conclude that providing partially known information can help us to improve estimation accuracy. But this shows only when the confidence region for the parameter becomes smaller than 3 times of the range defined by the posterior standard deviation. In terms of improvement ratio, it depends on the dependence between parameters. The more they depend from each other the higher the improvement ratio becomes. For future work, it will be interesting to analyze the influence of nuisance parameters on flaw parameter estimation, for example, the lift-off of the coil which is often known roughly from manual measurement.

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